

Tuned Two-Higgs-Doublet Model

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Abstract. We consider a Two-Higgs-Doublet Model (2HDM) constrained by the condition that assures cancellation of quadratic divergences up to the leading two-loop order. Regions in the parameter space consistent with existing experimental constraints and with the cancellation condition are determined. The possibility for CP violation in the scalar potential is discussed and regions of $\tan\beta - M_{H^\pm}$ with substantial amount of CP violation are found. The model allows to ameliorate the little hierarchy problem by lifting the minimal scalar Higgs boson mass and by suppressing the quadratic corrections to scalar masses. The cutoff originating from the naturalness arguments is therefore lifted from ~ 0.6 TeV in the Standard Model to $\gtrsim 2.5$ TeV in the 2HDM, depending on the mass of the lightest scalar.

1. Introduction

This project aims at extending the Standard Model (SM) in such a way that there would be no quadratic divergences up to the leading order at the two-loop level of the perturbation expansion. The quadratic divergences were first discussed within the SM by Veltman [1], who, adopting dimensional reduction [2], found the following quadratically divergent one-loop contribution to the Higgs boson (h) mass

$$\delta^{(\text{SM})}m_h^2 = \frac{\Lambda^2}{\pi^2 v^2} \left[\frac{3}{2}m_t^2 - \frac{1}{8}(6m_W^2 + 3m_Z^2) - \frac{3}{8}m_h^2 \right], \quad (1)$$

where Λ is a UV cutoff and $v \simeq 246$ GeV denotes the vacuum expectation value of the scalar doublet. The issue of quadratic divergences was then investigated further in [3] and [4].

Within the SM precision measurements require a light Higgs boson, therefore the correction (1) exceeds the mass itself even for small values of Λ , e.g. for $m_h = 130$ GeV one obtains $\delta^{(\text{SM})}m_h^2 \simeq m_h^2$ already for $\Lambda \simeq 600$ GeV. On the other hand, if we assume that the scale of new physics is widely separated from the electro-weak scale, then constraints that emerge from analysis of operators of dimension 6 require $\Lambda \gtrsim$ a few TeV. The lesson from this observation is that regardless of what physics lies beyond the SM, some amount of fine tuning is necessary; either we tune to lift the cutoff above $\Lambda \simeq 600$ GeV, or we tune when precision observables measured at LEP are fitted. Tuning both in corrections to the Higgs mass and in LEP physics is, of course, also a viable alternative which we are going to explore below. So, we will look for new physics in the TeV range which will allow to lift the cutoff implied by quadratic corrections to

m_h^2 to the multi-TeV range *and* which will be consistent with all the experimental constraints—both require some amount of tuning. Note that within the SM the requirement $\delta^{(\text{SM})}m_h^2 = 0$ implies an unrealistic value of the Higgs boson mass $m_h \simeq 310$ GeV.

Here we are going to argue that the Two-Higgs-Doublet Model (2HDM) in certain region of its parameter space can soften the little hierarchy problem both by suppressing quadratic corrections to scalar masses *and* it allows to lift the central value for the lightest Higgs mass.

2. The Two-Higgs-Doublet Model

In order to accommodate CP violation we consider here a 2HDM with softly broken \mathbb{Z}_2 symmetry which acts as $\Phi_1 \rightarrow -\Phi_1$ and $u_R \rightarrow -u_R$ (all other fields are neutral). The scalar potential then reads

$$\begin{aligned} V(\phi_1, \phi_2) = & -\frac{1}{2} \left\{ m_{11}^2 \phi_1^\dagger \phi_1 + m_{22}^2 \phi_2^\dagger \phi_2 + \left[m_{12}^2 \phi_1^\dagger \phi_2 + \text{H.c.} \right] \right\} + \frac{1}{2} \lambda_1 (\phi_1^\dagger \phi_1)^2 + \frac{1}{2} \lambda_2 (\phi_2^\dagger \phi_2)^2 \\ & + \lambda_3 (\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) + \lambda_4 (\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_1) + \frac{1}{2} \left[\lambda_5 (\phi_1^\dagger \phi_2)^2 + \text{H.c.} \right] \end{aligned} \quad (2)$$

The minimization conditions at $\langle \phi_1^0 \rangle = v_1/\sqrt{2}$ and $\langle \phi_2^0 \rangle = v_2/\sqrt{2}$ can be formulated as follows:

$$m_{11}^2 = v_1^2 \lambda_1 + v_2^2 (\lambda_{345} - 2\nu), \quad m_{22}^2 = v_2^2 \lambda_2 + v_1^2 (\lambda_{345} - 2\nu), \quad (3)$$

where $\lambda_{345} \equiv \lambda_3 + \lambda_4 + \text{Re } \lambda_5$ and $\nu \equiv \text{Re } m_{12}^2 / (2v_1 v_2)$.

2.1. Quadratic divergences

At the one-loop level the cancellation of quadratic divergences for the scalar Green's functions at zero external momenta (Γ_i , $i = 1, 2$) in the 2HDM type II model implies [5]

$$\Gamma_1 \equiv \frac{3}{2} m_W^2 + \frac{3}{4} m_Z^2 + \frac{v^2}{2} \left(\frac{3}{2} \lambda_1 + \lambda_3 + \frac{1}{2} \lambda_4 \right) - 3 \frac{m_b^2}{c_\beta^2} = 0, \quad (4)$$

$$\Gamma_2 \equiv \frac{3}{2} m_W^2 + \frac{3}{4} m_Z^2 + \frac{v^2}{2} \left(\frac{3}{2} \lambda_2 + \lambda_3 + \frac{1}{2} \lambda_4 \right) - 3 \frac{m_t^2}{s_\beta^2} = 0, \quad (5)$$

where $v^2 \equiv v_1^2 + v_2^2$, $\tan \beta \equiv v_2/v_1$ and we use the notation: $s_\theta \equiv \sin \theta$ and $c_\theta \equiv \cos \theta$. Note that when $\tan \beta$ is large, the two quark contributions can be comparable. In the type II model the mixed, $\phi_1 - \phi_2$, Green's function is not quadratically divergent.

The quartic couplings λ_i can be expressed in terms of the mass parameters and elements of the rotation matrix needed for diagonalization of the scalar masses (see, for example, Eqs. (3.1)–(3.5) of [6]). Therefore, for a given choice of α_i 's, the squared neutral-Higgs masses M_1^2 , M_2^2 and M_3^2 can be determined from the cancellation conditions (4)–(5) in terms of $\tan \beta$, μ^2 and $M_{H^\pm}^2$. It is worth noticing that scalar masses resulting from a scan over α_i , M_{H^\pm} and $\tan \beta$ exhibit a striking mass degeneracy in the case of large $\tan \beta$: $M_1 \simeq M_2 \simeq M_3 \simeq \mu^2 + 4m_b^2$.

At the two-loop level the leading contributions to quadratic divergences are of the form of $\Lambda^2 \ln \Lambda$. They could be determined adopting a method noticed by Einhorn and Jones [4], so that the cancellation conditions for quadratic divergences up to the leading two-loop order read:

$$\Gamma_1 + \delta\Gamma_1 = 0 \quad \text{and} \quad \Gamma_2 + \delta\Gamma_2 = 0 \quad (6)$$

with

$$\delta\Gamma_1 = \frac{v^2}{8} [9g_2\beta_{g_2} + 3g_1\beta_{g_1} + 6\beta_{\lambda_1} + 4\beta_{\lambda_3} + 2\beta_{\lambda_4}] \ln \left(\frac{\Lambda}{\bar{\mu}} \right) \quad (7)$$

$$\delta\Gamma_2 = \frac{v^2}{8} [9g_2\beta_{g_2} + 3g_1\beta_{g_1} + 6\beta_{\lambda_2} + 4\beta_{\lambda_3} + 2\beta_{\lambda_4} - 24g_t\beta_{g_t}] \ln \left(\frac{\Lambda}{\bar{\mu}} \right) \quad (8)$$

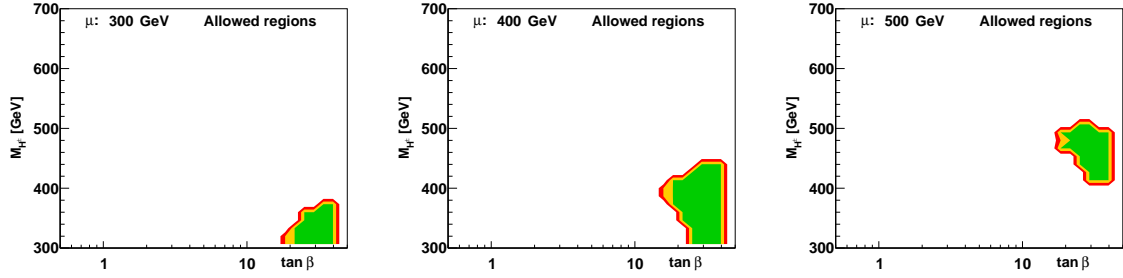


Figure 1. Two-loop allowed regions in the $\tan\beta$ - M_{H^\pm} plane, for $\Lambda = 2.5$ TeV, for $\mu = 300, 400, 500$ GeV (as indicated). Red: positivity is satisfied; yellow: positivity and unitarity both satisfied; green: also experimental constraints satisfied at the 95% C.L., as specified in the text.

where β 's are the appropriate beta functions while $\bar{\mu}$ is the renormalization scale. Hereafter we will be solving the conditions (6) for the scalar masses M_i^2 for a given set of α_i 's, $\tan\beta$, μ^2 and $M_{H^\pm}^2$. For the renormalization scale we will adopt v , so $\bar{\mu} = v$. Then those masses together with the corresponding coupling constants, will be used to find predictions of the model for various observables which then can be checked against experimental data.

2.2. Allowed regions

In order to find phenomenologically acceptable regions in the parameter space we impose the following experimental constraints: the oblique parameters T and S , $B_0 - \bar{B}_0$ mixing, $B \rightarrow X_s \gamma$, $B \rightarrow \tau \bar{\nu}_\tau X$, $B \rightarrow D \tau \bar{\nu}_\tau$, LEP2 Higgs-boson non-discovery, R_b , the muon anomalous magnetic moment and the electron electric dipole moment (for details concerning the experimental constraints, see refs. [7, 6, 8]). Subject to all these constraints, we find allowed solutions of (6). For instance, imposing all the experimental constraints we find allowed regions in the $\tan\beta$ - M_{H^\pm} plane as illustrated by the red domains in the $\tan\beta$ - M_{H^\pm} plane, see Fig. 1 for fixed values of μ . The allowed regions were obtained scanning over the mixing angles α_i and solving the two-loop cancellation conditions (6). Imposing also unitarity in the Higgs-Higgs-scattering sector [9, 10, 11] (yellow regions), the allowed regions are only slightly reduced. Requiring that also experimental constraints are satisfied the green regions are obtained.

For parameters that are consistent with unitarity, positivity, experimental constraints and the two-loop cancellation conditions (6), we show in Fig. 2 scalar masses resulting from a scan over α_i , M_{H^\pm} and $\tan\beta$. As we have noticed for the one-loop spectrum, large $\tan\beta$ implies similar scalar masses. This is indeed what is being observed in Fig. 2 also for the two-loop case. The allowed solutions “peak” around $M_{H^\pm} \sim \mu$ with $20 \lesssim \tan\beta \lesssim 50$.

2.3. CP violation

Here we will verify the possibility of having CP violation in the scalar potential (2), subject to the two-loop cancellation of quadratic divergences (6). In order to parametrize the magnitude of CP violation we adopt the $U(2)$ -invariants introduced by Lavoura and Silva [12] (see also [13]). However here we use the basis-invariant formulation of these invariants J_1 , J_2 and J_3 as proposed by Gunion and Haber [14]. As is proven there (theorem #4) the Higgs sector is

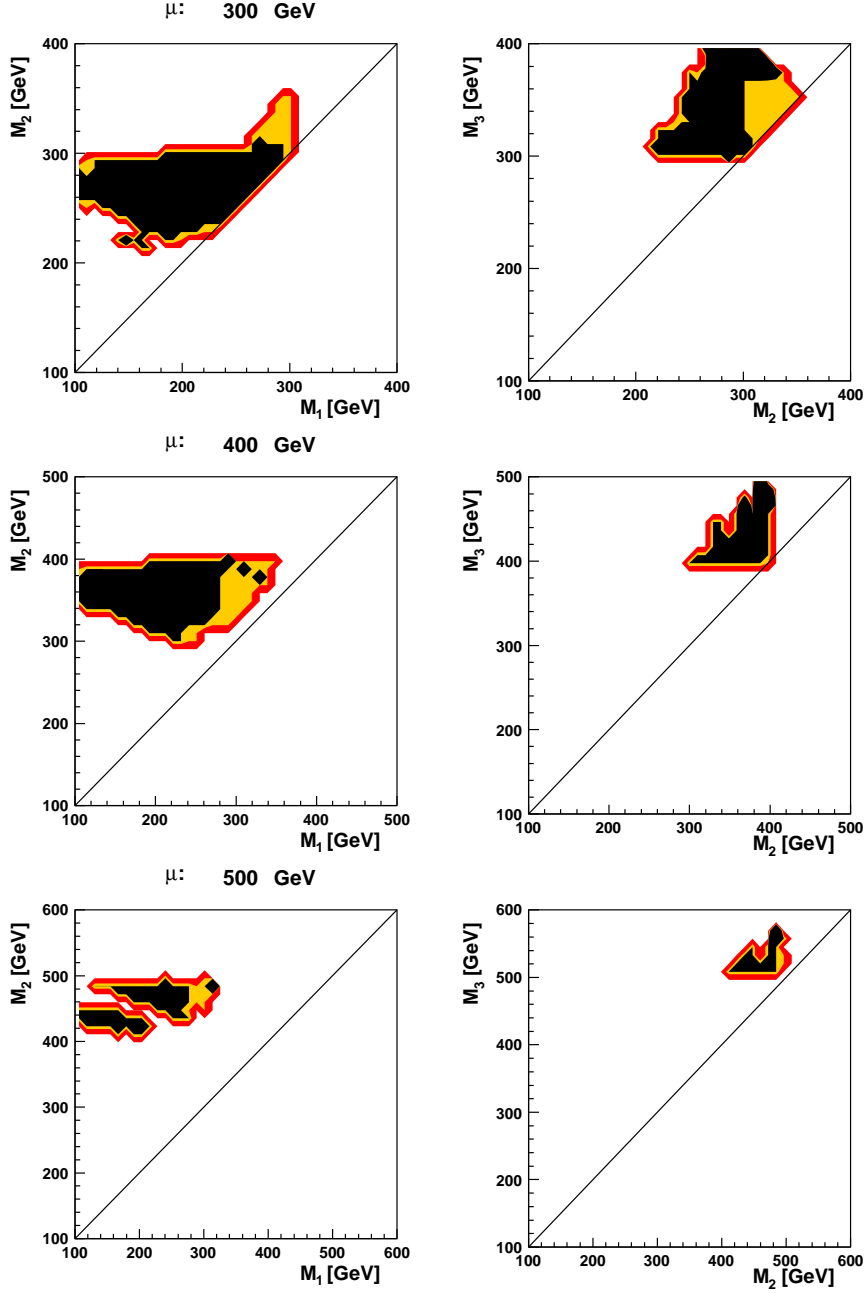


Figure 2. Two-loop distributions of allowed masses M_2 vs M_1 (left panels) and M_3 vs M_2 (right) for $\Lambda = 2.5$ TeV, resulting from a scan over the full range of α_i , $\tan \beta \in (0.5, 50)$ and $M_{H^\pm} \in (300, 700)$ GeV, for $\mu = 300, 400, 500$ GeV. Red: Positivity is satisfied; yellow: positivity and unitarity both satisfied; green: also experimental constraints satisfied at the 95% C.L., as specified in the text.

CP-conserving if and only if $\text{Im } J_i = 0$ for all i . In the basis adopted here the invariants read [7]:

$$\text{Im } J_1 = -\frac{v_1^2 v_2^2}{v^4} (\lambda_1 - \lambda_2) \text{Im } \lambda_5, \quad (9)$$

$$\begin{aligned} \text{Im } J_2 = -\frac{v_1^2 v_2^2}{v^8} [& ((\lambda_1 - \lambda_3 - \lambda_4)^2 - |\lambda_5|^2) v_1^4 + 2(\lambda_1 - \lambda_2) \text{Re } \lambda_5 v_1^2 v_2^2 \\ & - ((\lambda_2 - \lambda_3 - \lambda_4)^2 - |\lambda_5|^2) v_2^4] \text{Im } \lambda_5, \end{aligned} \quad (10)$$

$$\text{Im } J_3 = \frac{v_1^2 v_2^2}{v^4} (\lambda_1 - \lambda_2) (\lambda_1 + \lambda_2 + 2\lambda_4) \text{Im } \lambda_5. \quad (11)$$

It is seen that there is no CP violation when $\text{Im } \lambda_5 = 0$, see [7] for more details.

As we have noted earlier, $\tan \beta$ above ~ 40 implies approximate degeneracy of scalar masses. That could jeopardize the CP violation in the potential since it is well known that the exact degeneracy $M_1 = M_2 = M_3$ results in vanishing invariants $\text{Im } J_i$ and no CP violation (exact degeneracy implies $\text{Im } \lambda_5 = 0$). Using the one-loop conditions (4)–(5) one immediately finds that $\lambda_1 - \lambda_2 = 4(m_b^2/c_\beta^2 - m_t^2/s_\beta^2)/v^2$, which implies

$$\text{Im } J_1 = 4 \text{Im } \lambda_5 (c_\beta^2 m_t^2 - s_\beta^2 m_b^2)/v^2 = -4 \text{Im } \lambda_5 (m_b/v)^2 + \mathcal{O}(\text{Im } \lambda_5 / \tan \beta^2) \quad (12)$$

Note that if $\tan \beta$ is large then $\text{Im } J_1$ is suppressed not only by $\text{Im } \lambda_5 \simeq 0$ (as caused by $M_1 \simeq M_2 \simeq M_3$) but also by the factor (m_b^2/v^2) , as implied by the cancellation conditions (4)–(5). The same suppression factor appears for $\text{Im } J_3$. The case of $\text{Im } J_2$ is more involved, however when m_b^2/v^2 is neglected all the invariants (9)–(11) have the same simple asymptotic behavior for large $\tan \beta$:

$$\text{Im } J_i \sim \text{Im } \lambda_5 / \tan^2 \beta \quad (13)$$

Those conclusions qualitatively remain also at the two-loop level. For a quantitative illustration we plot in Fig. 3 maximal values of the invariants in the $\tan \beta$ – M_{H^\pm} plane with all the necessary constraints imposed, looking for regions which still allow for substantial CP violation. At high values of $\tan \beta$ these invariants are of the order of 10^{-3} , in qualitative agreement with the discussion above. It is worth noticing that the corresponding invariant in the SM; $\text{Im } Q = \text{Im}(V_{ud}V_{cb}V_{ub}^*V_{cd}^*)$ [15] is of the order of $\sim 2 \times 10^{-5} \sin \delta_{KM}$ (V_{ij} and δ_{KM} are elements of the CKM matrix and CP-violating phase, respectively). Therefore the model considered here allows for CP violation at least two orders of magnitude larger than in the SM.

3. Summary

The goal of this project was to build a minimal realistic model which would ameliorate the little hierarchy problem through suppression of the quadratic divergences in scalar boson mass corrections and through lifting the mass of the lightest Higgs boson. It has been shown that it could be accomplished within the Two-Higgs-Doublet Model type II. Phenomenological consequences of requiring no quadratic divergences in corrections to scalar masses were discussed. The 2HDM type II was analyzed taking into account the relevant existing experimental constraints. Allowed regions in the parameter space were determined. An interesting scalar mass degeneracy was noticed for $\tan \beta \gtrsim 40$. The issue of possible CP violation in the scalar potential was discussed and regions of $\tan \beta - M_{H^\pm}$ with substantial strength of CP violation were identified. The cutoff implied by the naturalness arguments is lifted from ~ 600 GeV in the SM up to at least $\gtrsim 2.5$ TeV, depending on the mass of the lightest scalar. In order to accommodate a possibility for dark matter a scalar gauge singlet should be added to the model.

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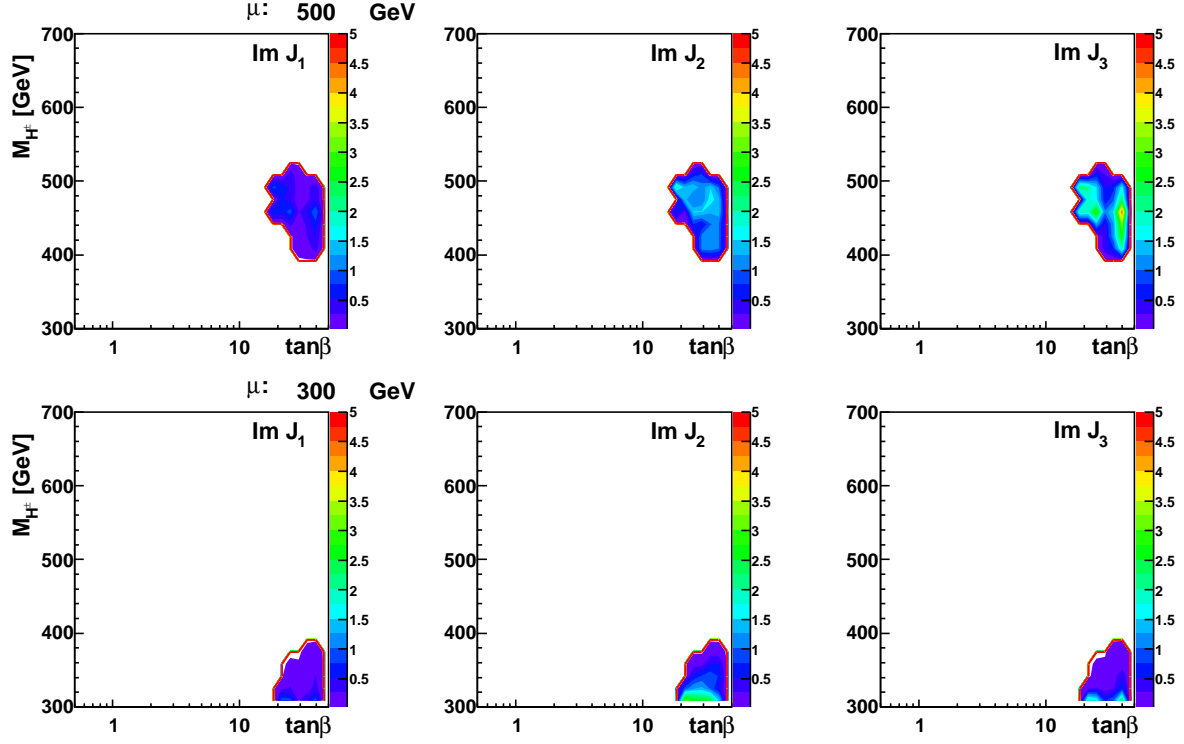


Figure 3. Absolute values of the imaginary parts of the $U(2)$ -invariants $|\text{Im } J_i|$ at the two-loop level for $\Lambda = 2.5$ TeV, for $\mu = 500$ GeV (top) and $\mu = 300$ GeV (bottom). The color coding in units 10^{-3} is given along the right vertical axis.

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